

AMCS / CS 247 – Scientific Visualization

Lecture 13: Volume Visualization, Pt. 2

Markus Hadwiger, KAUST

Reading Assignment #7 (until Oct 19)



Read (required):

- Real-Time Volume Graphics, Chapter 3.2.3 (*Compositing*)
- Real-Time Volume Graphics, Chapters 5.5, 5.6 (*Gradients*)
- Real-Time Volume Graphics, Chapter 7 (*GPU-Based Ray Casting*)

Read (optional):

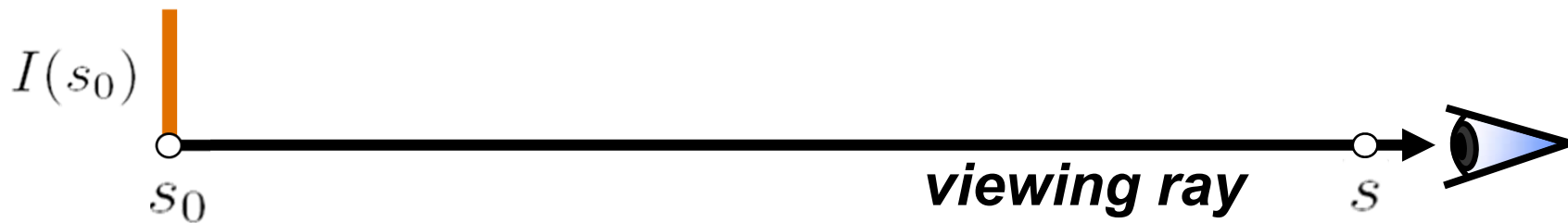
- Paper:
Nelson Max, Optical Models for Direct Volume Rendering,
IEEE Transactions on Visualization and Computer Graphics, 1995
<http://dx.doi.org/10.1109/2945.468400>

Volume Rendering Integral



How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



Initial intensity
at s_0

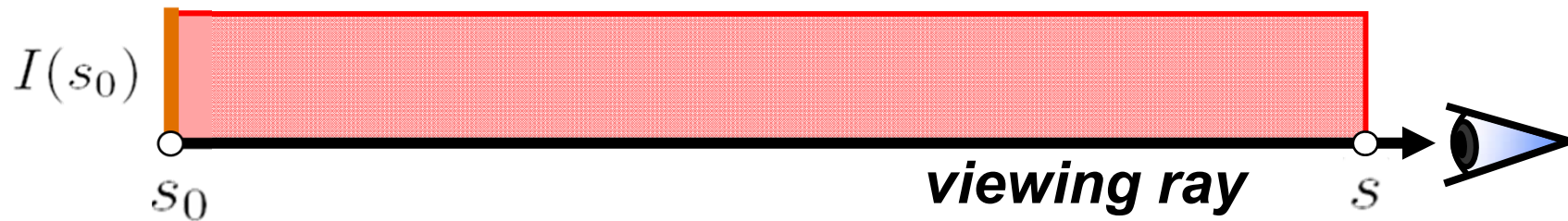
$$I(s) = I(s_0)$$

Volume Rendering Integral



How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



Initial intensity
at s_0

$$I(s) = I(s_0)$$

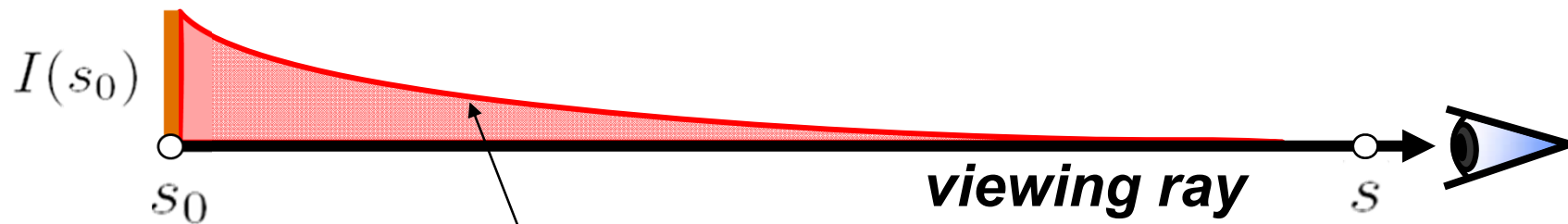
Without absorption all
the initial radiant energy
would reach the point s .

Volume Rendering Integral



How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



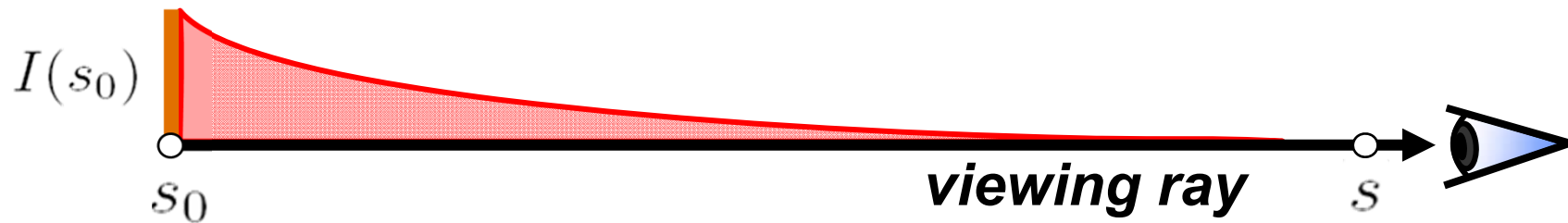
$$I(s) = I(s_0) e^{-\tau(s_0, s)}$$

Volume Rendering Integral



How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



Optical depth τ
Absorption κ

$$I(s) = I(s_0) e^{-\tau(s_0, s)}$$

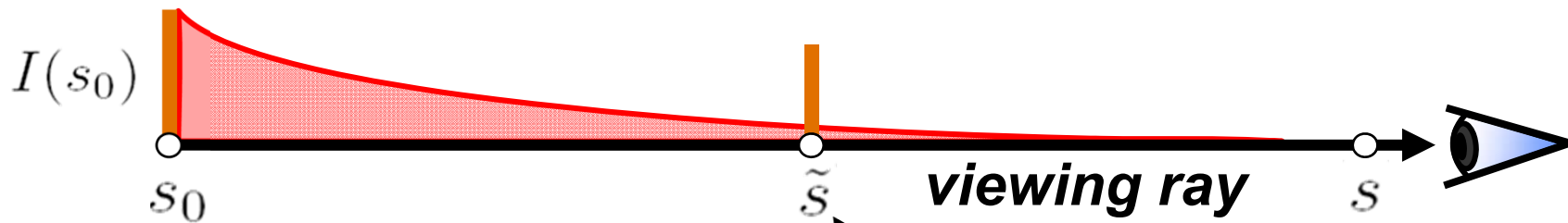
$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s) ds.$$

Volume Rendering Integral



How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



One point \tilde{s} along the viewing ray emits additional radiant energy.

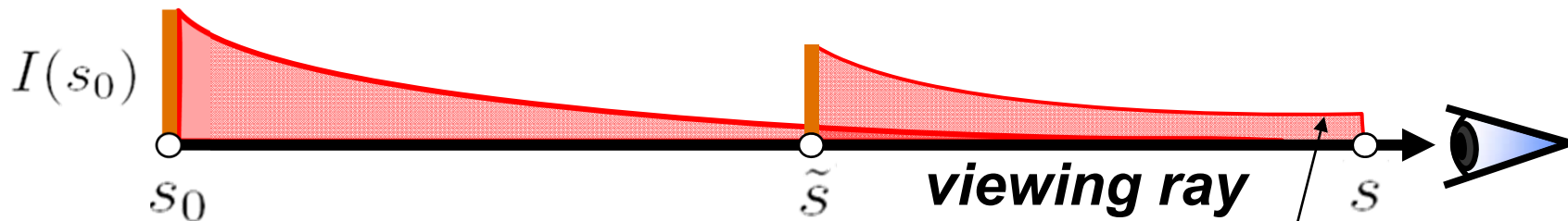
$$I(s) = I(s_0) e^{-\tau(s_0,s)} + q(\tilde{s})$$

Volume Rendering Integral



How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



One point \tilde{s} along the viewing ray emits additional radiant energy.

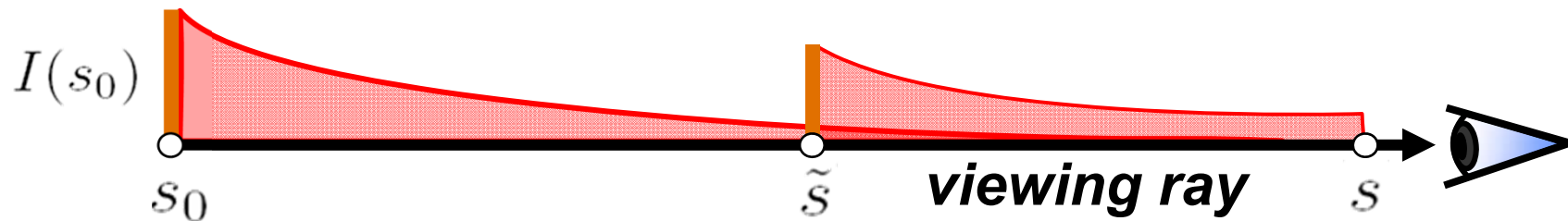
$$I(s) = I(s_0) e^{-\tau(s_0,s)} + q(\tilde{s}) e^{-\tau(\tilde{s},s)}$$

Volume Rendering Integral



How do we determine the radiant energy along the ray?

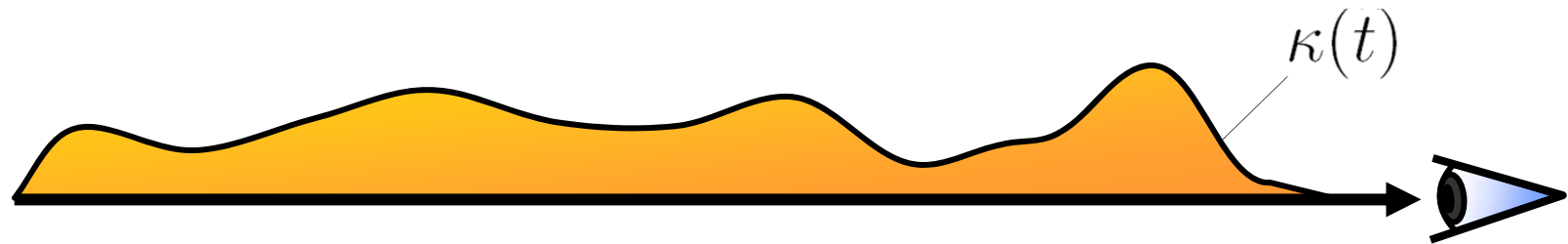
Physical model: emission and absorption, no scattering



Every point \tilde{s} along the viewing ray emits additional radiant energy

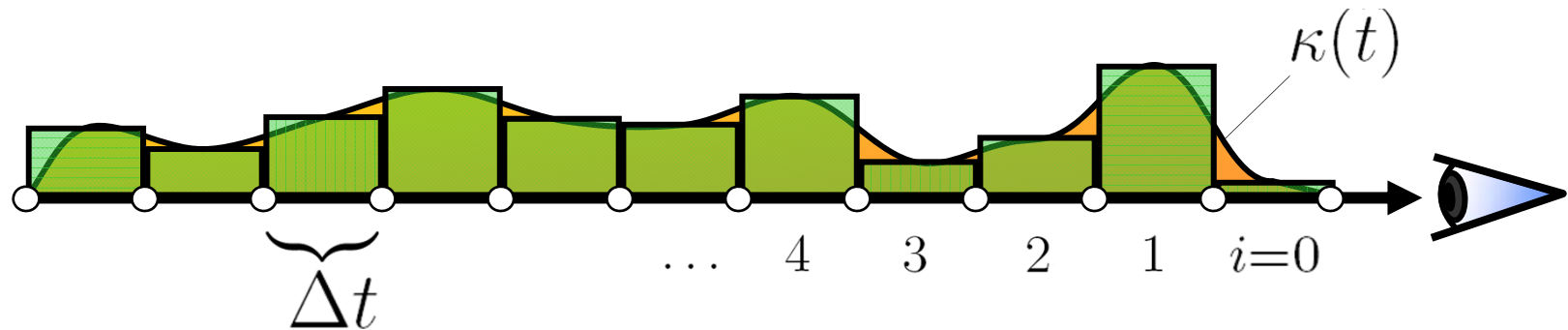
$$I(s) = I(s_0) e^{-\tau(s_0, s)} + \int_{s_0}^s q(\tilde{s}) e^{-\tau(\tilde{s}, s)} d\tilde{s}$$

Volume Rendering Integral: Numerical Solution



Optical depth: $\tau(0, t) = \int_0^t \kappa(\hat{t}) d\hat{t}$

Volume Rendering Integral: Numerical Solution

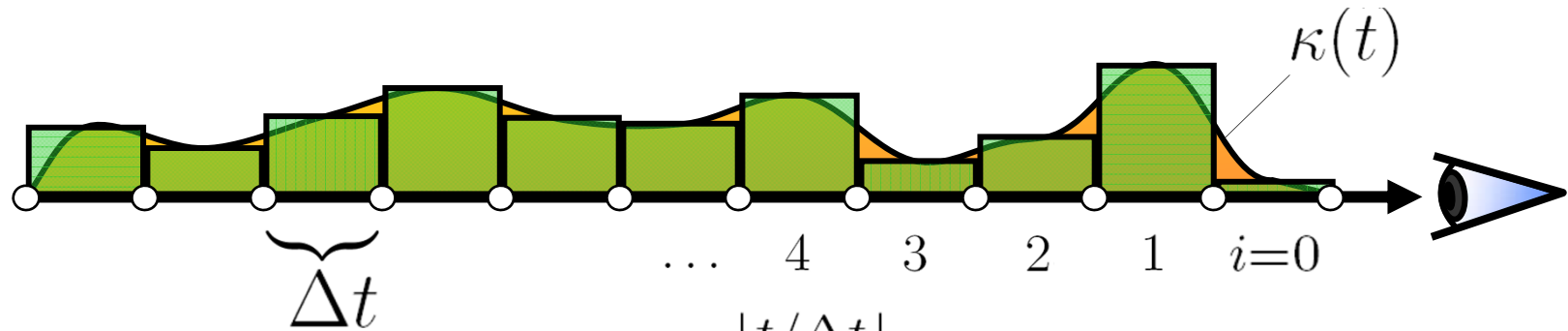


Optical depth: $\tau(0, t) = \int_0^t \kappa(\hat{t}) d\hat{t}$

Approximate Integral by Riemann sum:

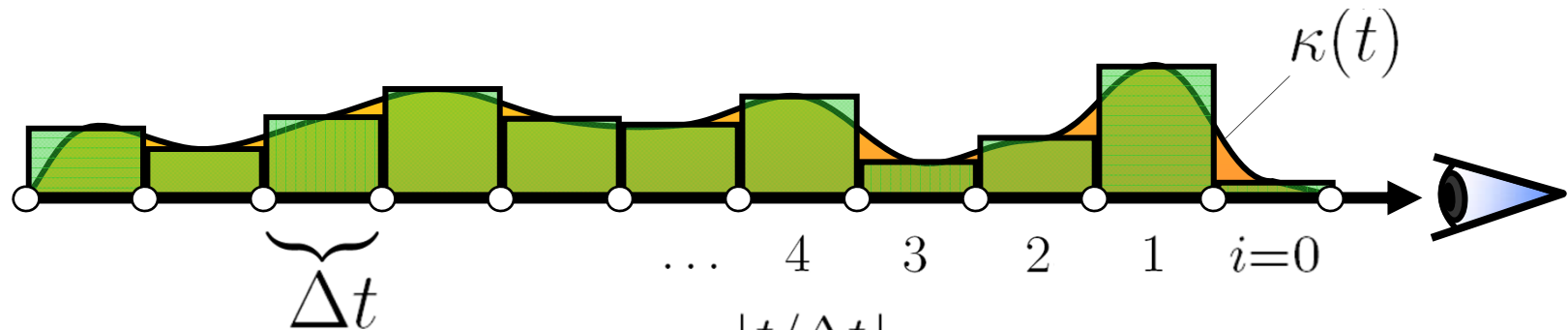
$$\tau(0, t) \approx \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

Volume Rendering Integral: Numerical Solution



$$\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

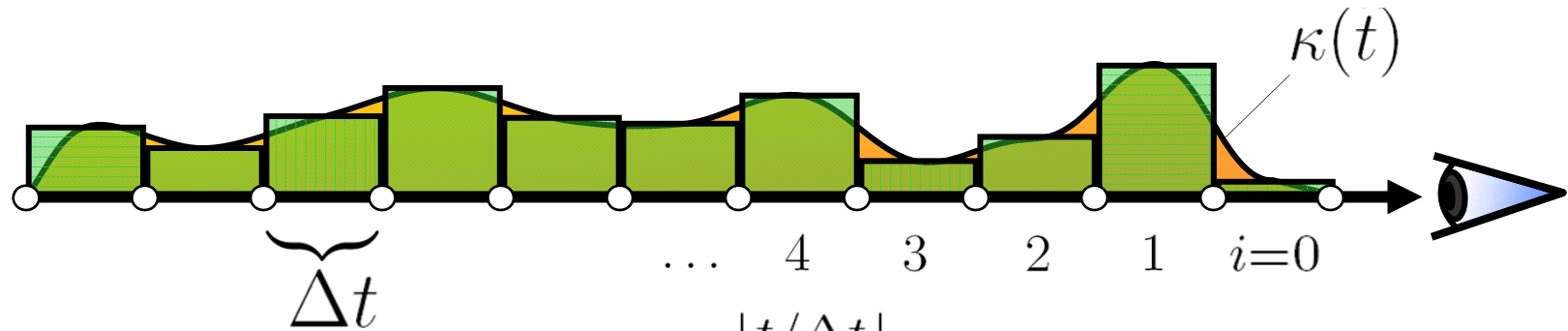
Volume Rendering Integral: Numerical Solution



$$\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

$$e^{-\tilde{\tau}(0, t)} = e^{-\sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t}$$

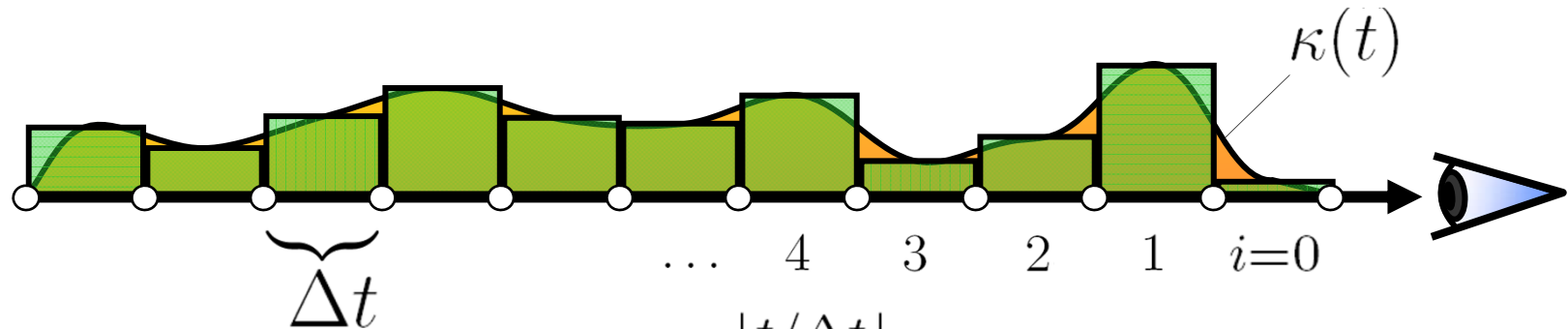
Volume Rendering Integral: Numerical Solution



$$\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

$$e^{-\tilde{\tau}(0, t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} e^{-\kappa(i \cdot \Delta t) \Delta t}$$

Volume Rendering Integral: Numerical Solution



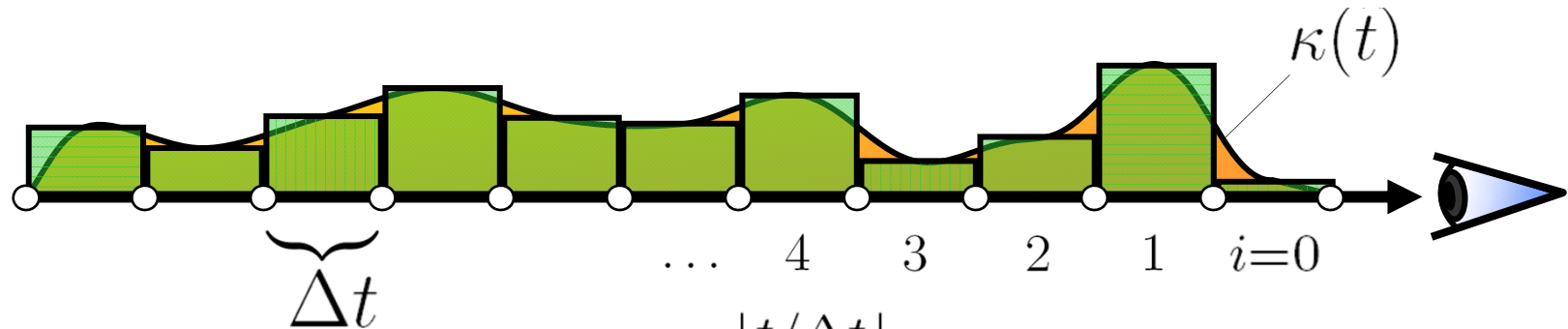
$$\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

$$e^{-\tilde{\tau}(0, t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} e^{-\kappa(i \cdot \Delta t) \Delta t}$$

Now we introduce *opacity*:

$$A_i = 1 - e^{-\kappa(i \cdot \Delta t) \Delta t}$$

Volume Rendering Integral: Numerical Solution



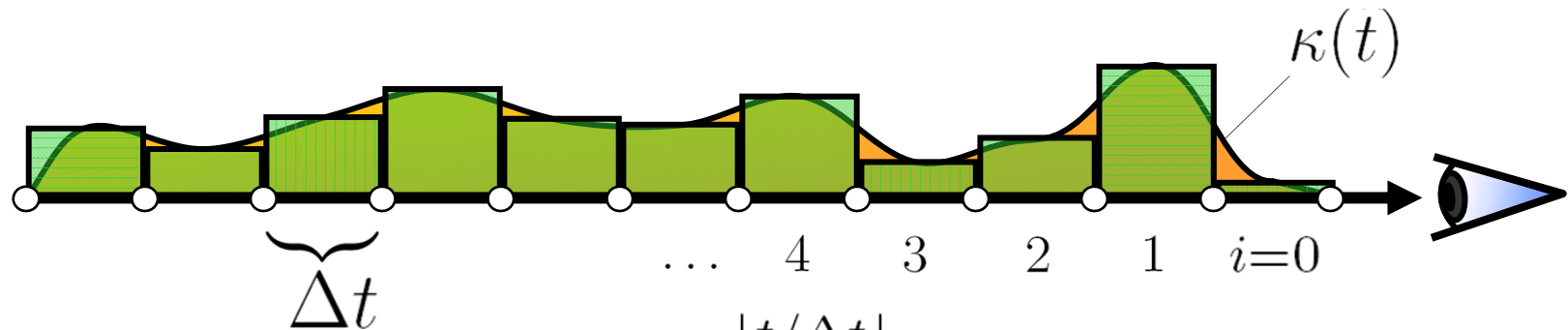
$$\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

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Now we introduce *opacity*:

$$1 - A_i = e^{-\kappa(i \cdot \Delta t) \Delta t}$$

Volume Rendering Integral: Numerical Solution



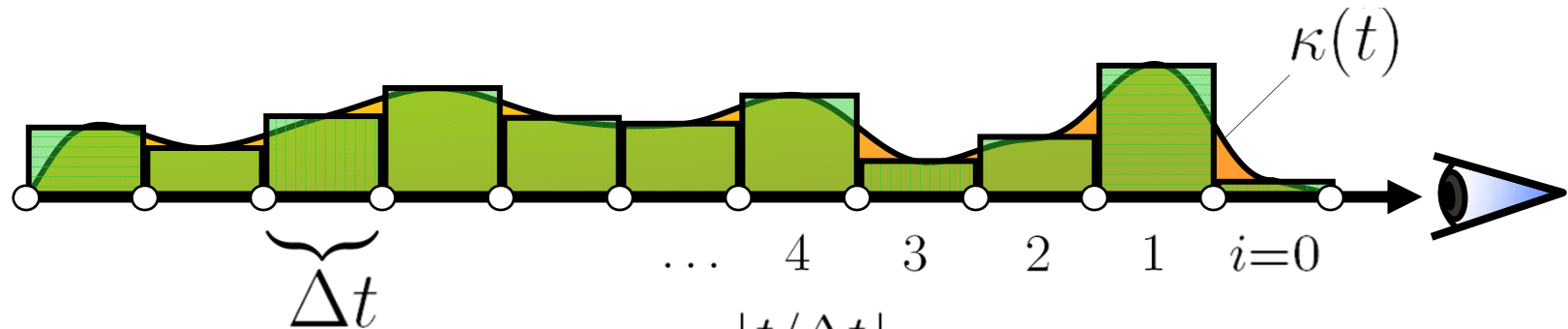
$$\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

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Volume Rendering Integral: Numerical Solution



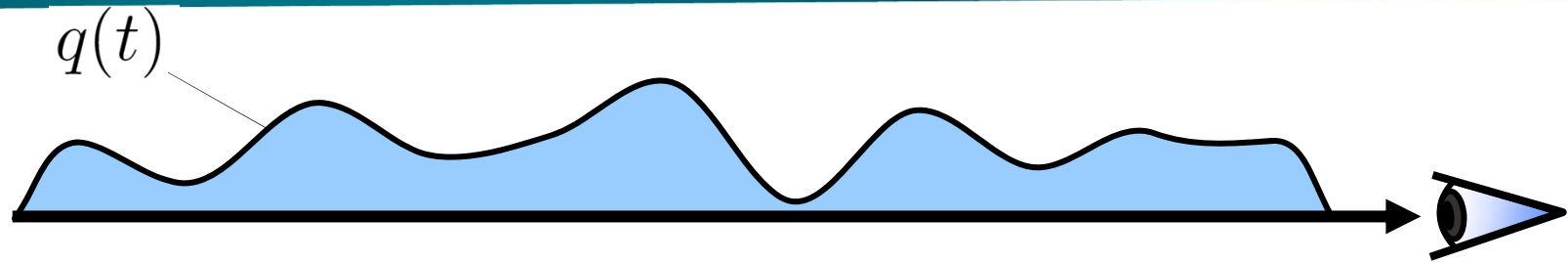
$$\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

$$e^{-\tilde{\tau}(0, t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

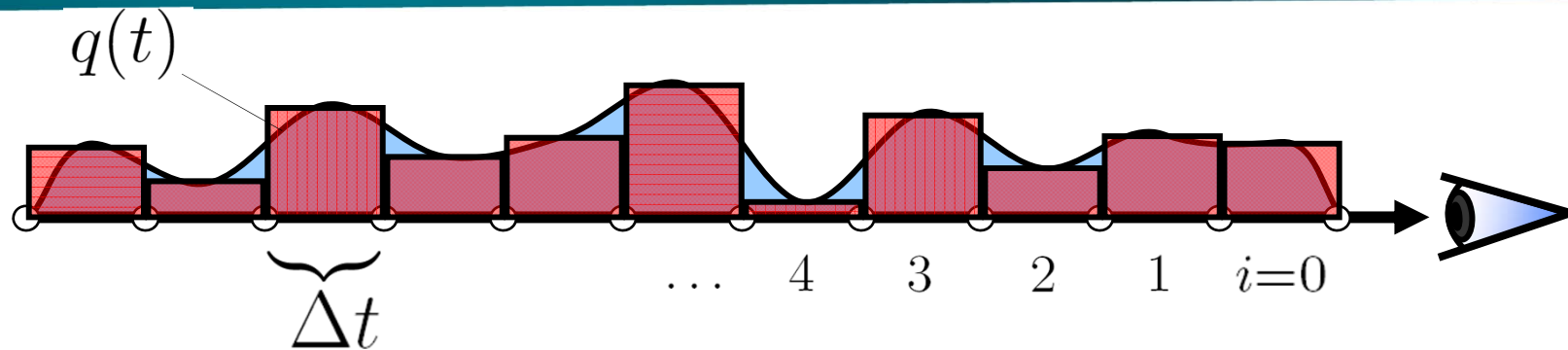
Now we introduce *opacity*:

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Volume Rendering Integral: Numerical Solution



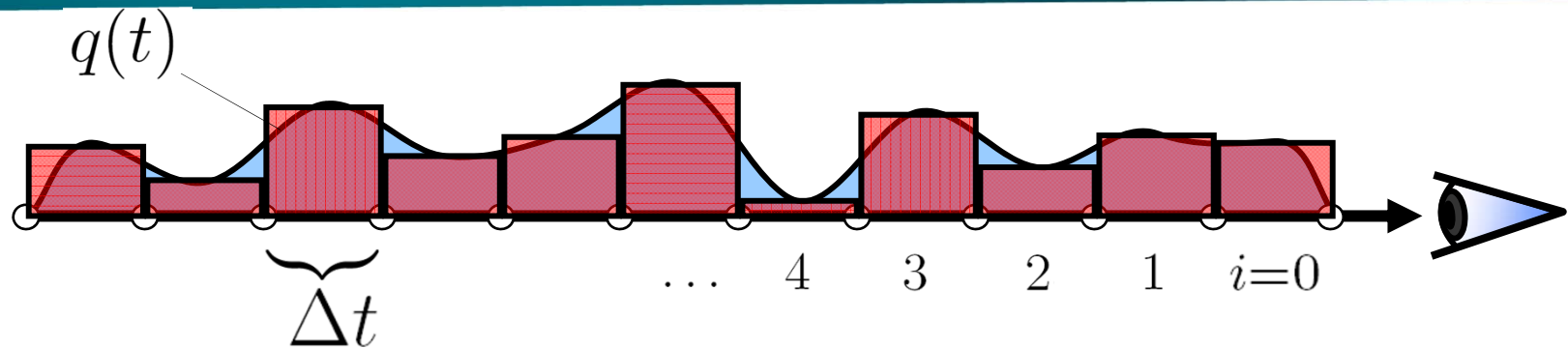
Volume Rendering Integral: Numerical Solution



$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t) \Delta t$$

Volume Rendering Integral: Numerical Solution

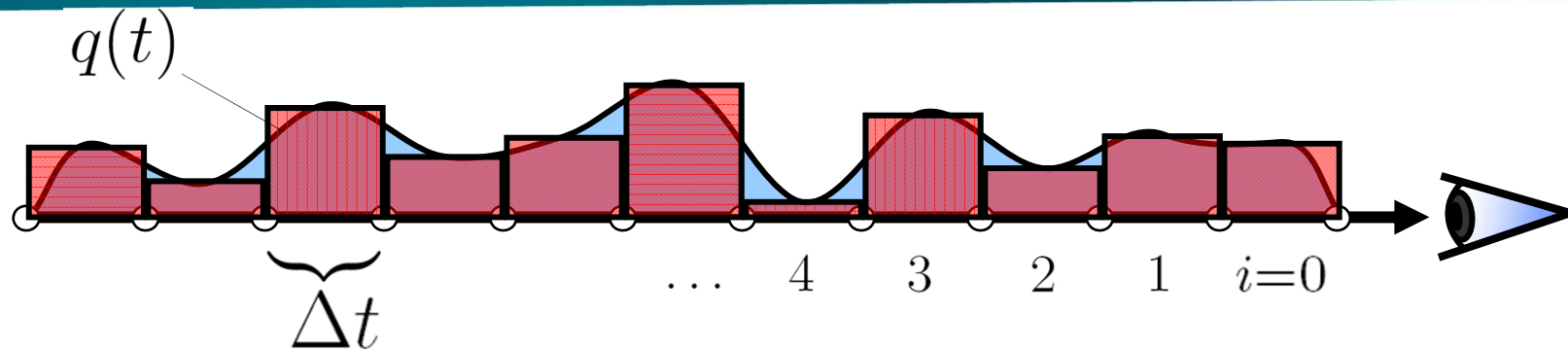


$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t) \Delta t$$

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i e^{-\tilde{\tau}(0,t)}$$

Volume Rendering Integral: Numerical Solution

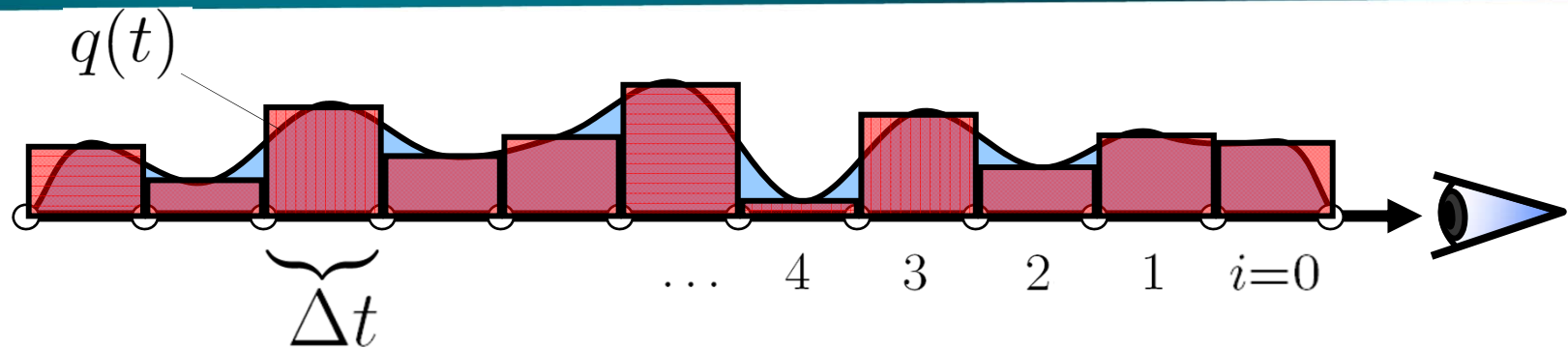


$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

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Volume Rendering Integral: Numerical Solution

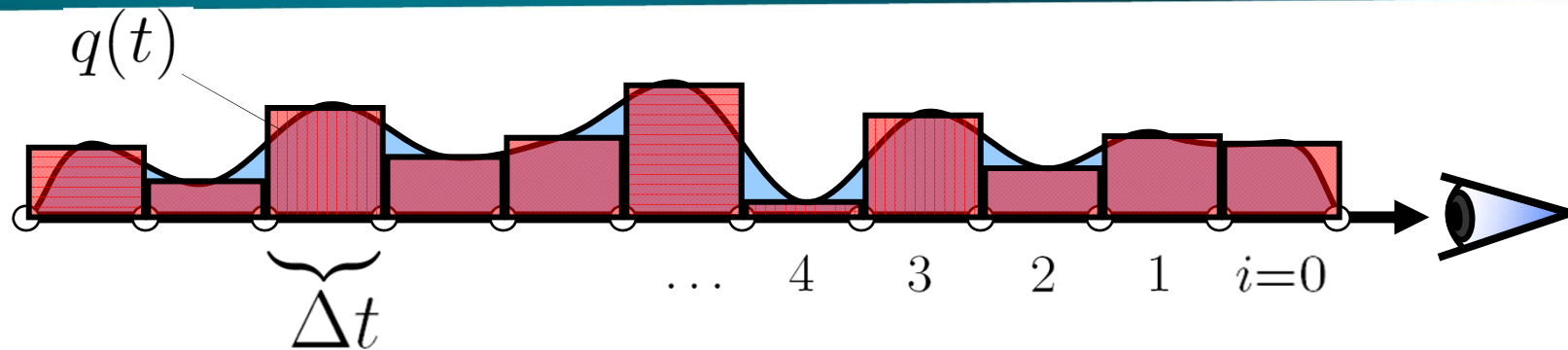


$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

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Volume Rendering Integral: Numerical Solution



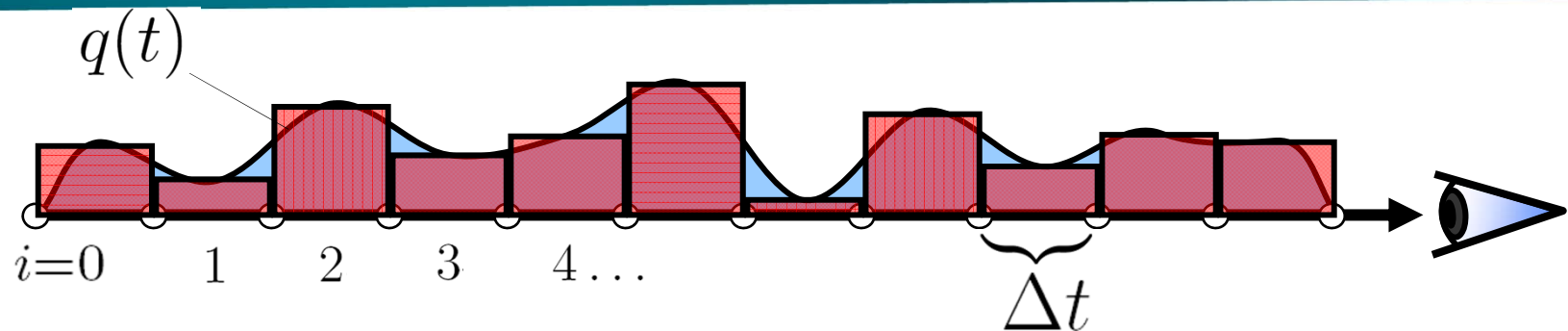
$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

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$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

can be computed recursively/iteratively!

Volume Rendering Integral: Numerical Solution

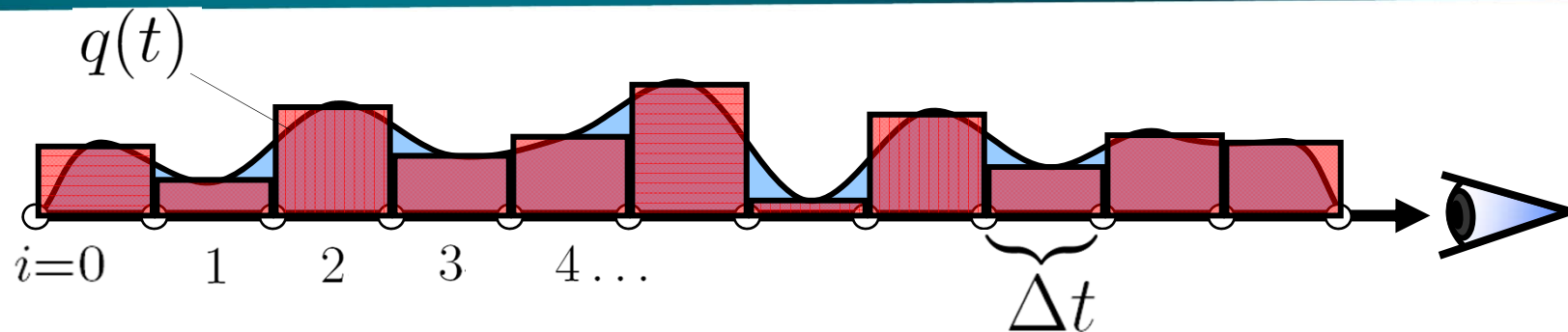


Note: we just changed the convention from $i=0$ is at the front of the volume (previous slides) to $i=0$ is at the back of the volume !

can be computed recursively/iteratively:

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Volume Rendering Integral: Numerical Solution



can be computed recursively/iteratively:

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

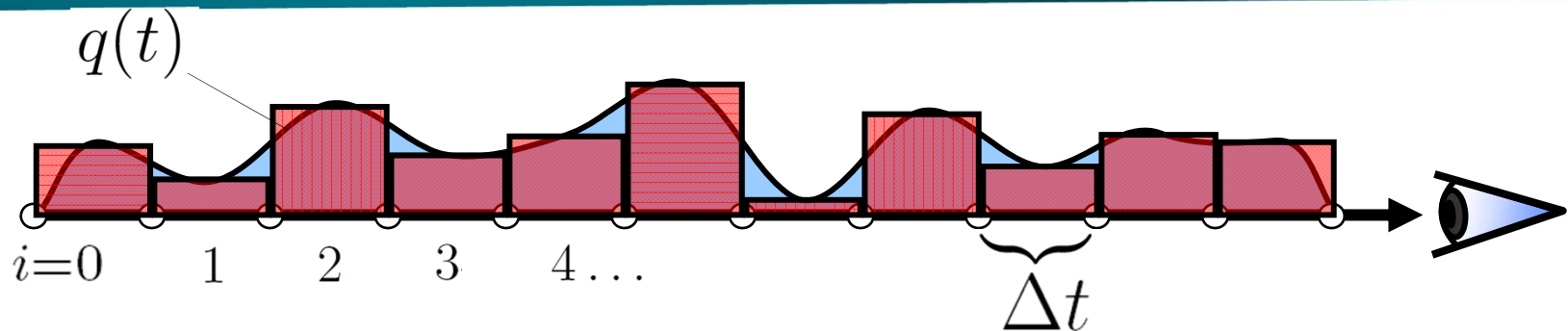
Radiant energy
observed at position i

Radiant energy
emitted at position i

Absorption at
position i

Radiant energy
observed at position $i-1$

Volume Rendering Integral: Numerical Solution



**Back-to-front
compositing**

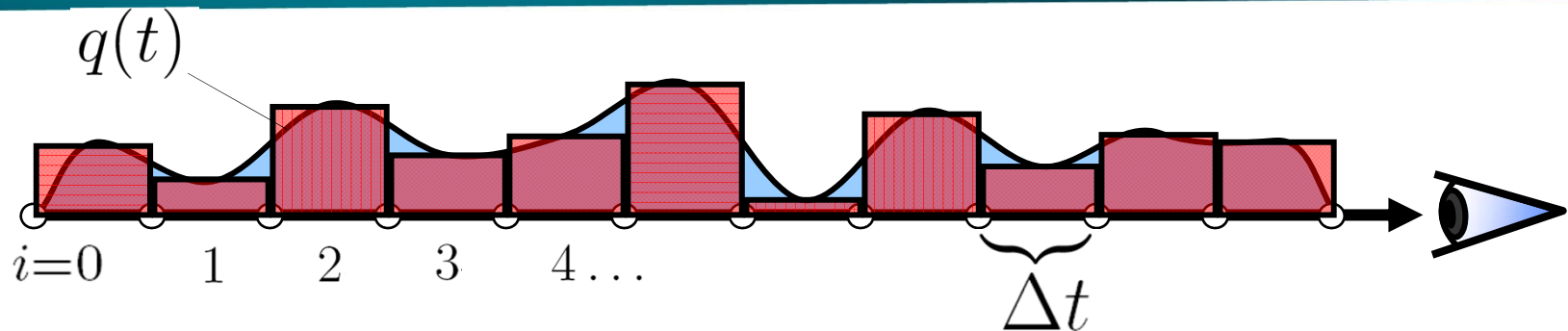
$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

**Front-to-back
compositing**

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$

$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

Volume Rendering Integral: Numerical Solution



**Back-to-front
compositing**

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Iterate from $i=0$ (back) to $i=\max$ (front): i increases

**Front-to-back
compositing**

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$

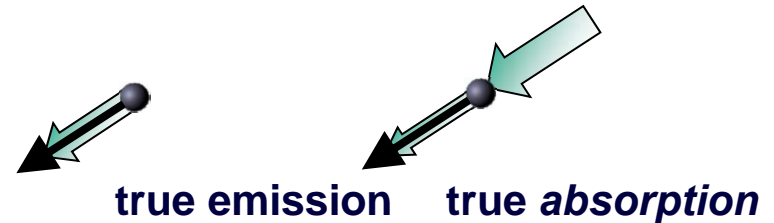
$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

Iterate from $i=\max$ (front) to $i=0$ (back) : i decreases

Volume Rendering Integral Summary



Volume rendering integral
for *Emission Absorption* model



$$I(s) = I(s_0) e^{-\tau(s_0, s)} + \int_{s_0}^s q(\tilde{s}) e^{-\tau(\tilde{s}, s)} d\tilde{s}$$

Numerical solutions:

Back-to-front compositing

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Front-to-back compositing

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$
$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama