King Abdullah University of Science and Technology KAUST

AMCS / CS 247 – Scientific Visualization
Lecture 24: Vector Field / Flow Visualization, Pt. 5

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Reading Assignment #12 (until Nov 23)

Read (required):

- Streak Lines as Tangent Curves of a Derived Vector Field,
  Tino Weinkauf and Holger Theisel, IEEE Vis 2010
  http://dx.doi.org/10.1109/TVCG.2010.198

Quiz #3: Nov 30

Organization

- First 30 min of lecture
- No material (book, notes, ...) allowed

Content of questions

- Lectures (both actual lectures and slides)
- Reading assignments (except optional ones)
- Programming assignments (algorithms, methods)
- Solve short practical examples

Vector Fields and Dynamical Systems (1)

Jacobian (**matrix**), (vector field → tensor field)

- Gradient of vector field: how fast do the vectors change?
- In our context: also called *velocity gradient tensor*

$$\mathbf{J} = \nabla \mathbf{v} \qquad \mathbf{J}(x,y,z) = \begin{pmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{pmatrix}$$

- Can be decomposed into *symmetric* part + *antisymmetric* part

$$\mathbf{J} = \mathbf{D} + \mathbf{S}$$ *velocity gradient tensor*

sym.: $\mathbf{D} = \frac{1}{2} (\mathbf{J} + \mathbf{J}^{T})$ strain: *rate-of-strain tensor*

skew-sym.: $\mathbf{S} = \frac{1}{2} (\mathbf{J} - \mathbf{J}^{T})$ rotation: *vorticity/spin tensor*

Vector Fields and Dynamical Systems (2)

Vorticity/spin/angular velocity tensor

- Antisymmetric part of velocity gradient tensor
- Corresponds to vorticity/curl/angular velocity (beware of factor ½)

$$\mathbf{S} = \tfrac{1}{2} (\mathbf{J} - \mathbf{J}^T)$$

$$\mathbf{S} = \tfrac{1}{2} \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \qquad \omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \nabla \times \mathbf{v} = \begin{pmatrix} w_y - v_z \\ u_z - w_x \\ v_x - u_y \end{pmatrix}$$

$\mathbf{S}$ acts on vector like cross product with ω: $\mathbf{S} \bullet = \tfrac{1}{2} \omega \times$

Vector Fields and Dynamical Systems (3)

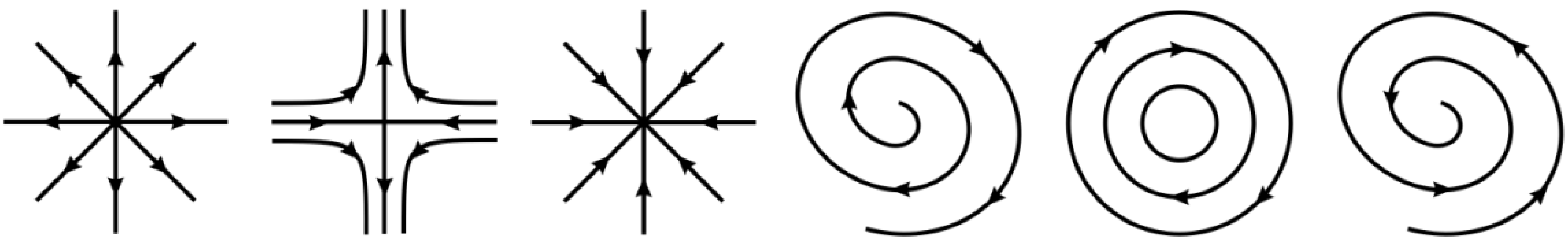
Critical point

- Velocity vanishes (all components zero) $\mathbf{v} = \mathbf{0}$

$$\mathbf{v}(\mathbf{x}_0) = \mathbf{0} \quad \text{with} \quad \mathbf{v}(\mathbf{x}_0 \pm \boldsymbol{\epsilon}) \neq \mathbf{0}$$

Characterize using the Jacobian $\mathbf{J}$ at a point $\mathbf{v} = \mathbf{0}$

- Look at eigenvalues (and eigenvectors) of $\mathbf{J}$ $\mathbf{J}$ $\det(\mathbf{J}(\mathbf{x}_0)) \neq 0$



Repelling node	Saddle point	Attracting node	Repelling focus	Center	Attracting focus
$R_1, R_2 > 0$	$R_1 < 0, R_2 > 0$	$R_1, R_2 < 0$	$R_1 = R_2 > 0$	$R_1 = R_2 = 0$	$R_1 = R_2 < 0$
$I_1 = I_2 = 0$	$I_1 = I_2 = 0$	$I_1 = I_2 = 0$	$I_1 = -I_2 \neq 0$	$I_1 = -I_2 \neq 0$	$I_1 = -I_2 \neq 0$

(Non-Linear) Dynamical Systems

Start with system of linear ODEs

- Non-linear systems can be linearized around a point
- Use linearization for characterization

$$\dot{\mathbf{x}} = A\mathbf{x} \qquad A \text{ is an } n \times n \text{ matrix}$$

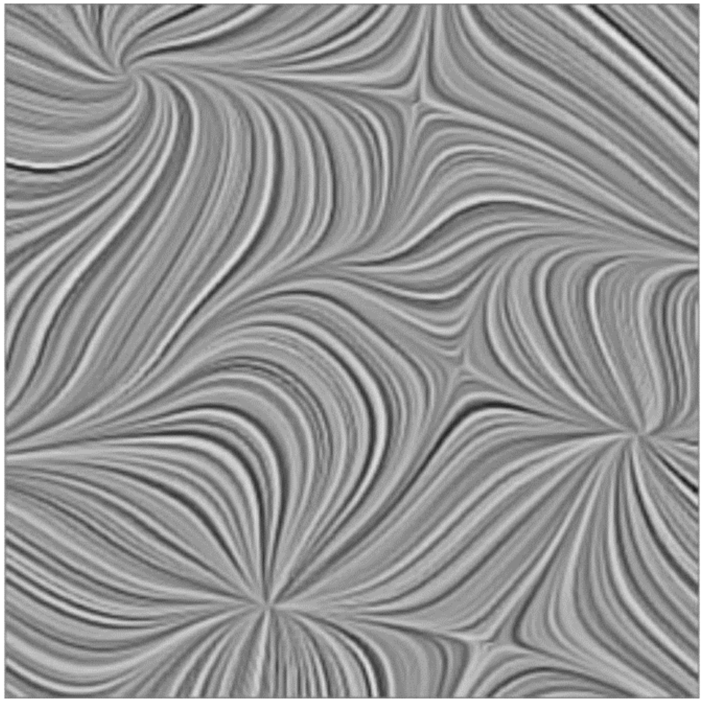
$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = \begin{bmatrix} \frac{dx_1}{dt} \\ \vdots \\ \frac{dx_n}{dt} \end{bmatrix}$$

$$\mathbf{x}(0) = \mathbf{x}_0$$

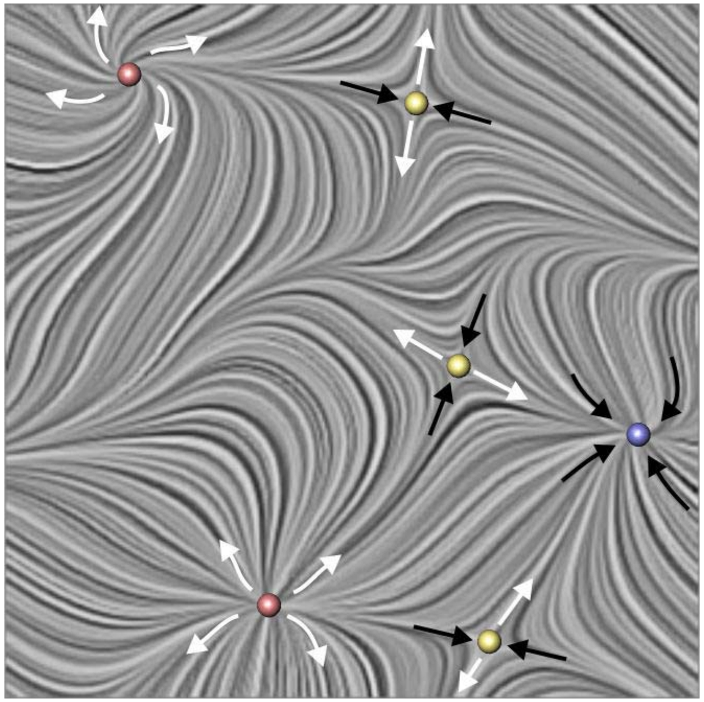
solution: $\mathbf{x}(t) = e^{At}\mathbf{x}_0$

characterize behavior
through eigenvalues of A

Critical Points

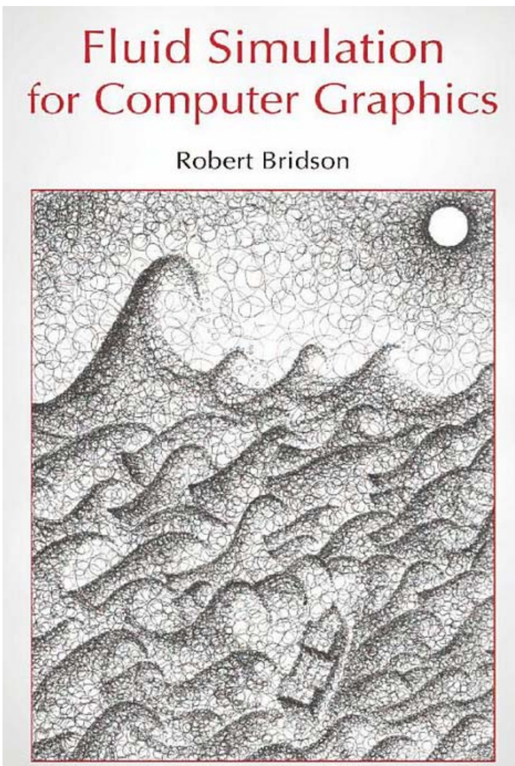


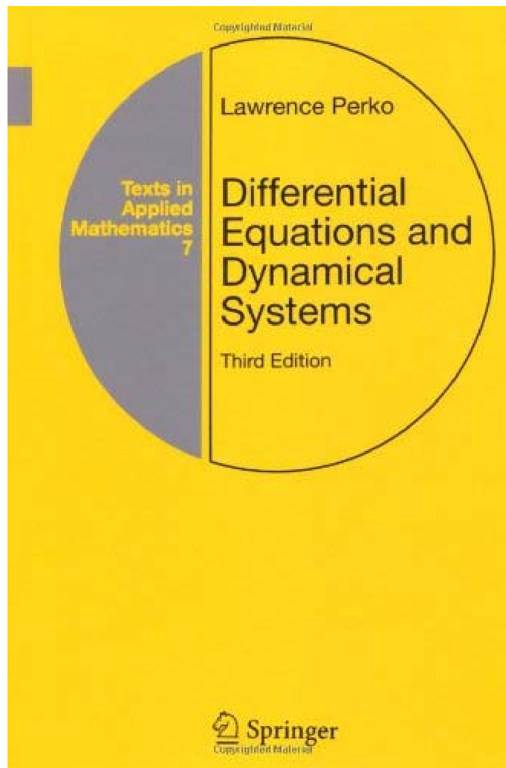
stream lines

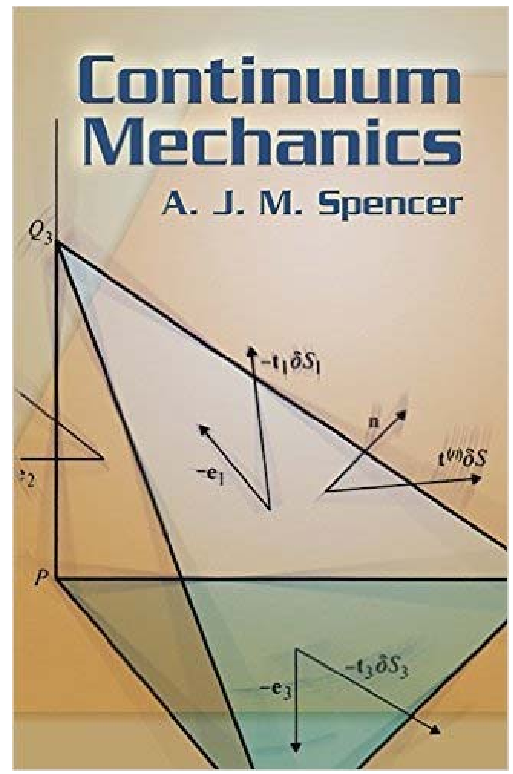


critical points

Recommended Books







Thank you.

Thanks for material

- Tino Weinkauf
- Holger Theisel
- Ronny Peikert
- Jens Krüger