King Abdullah University of Science and Technology KAUST

AMCS / CS 247 – Scientific Visualization
Lecture 26: Vector Field / Flow Visualization, Pt. 6

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Reading Assignment #13 (until May 1)

Read (required):

- B. Cabral and C. Leedom: *Imaging Vector Fields Using Line Integral Convolution*, SIGGRAPH 1993

  `http://dx.doi.org/10.1145/166117.166151`

- Learn how convolution (the convolution of two functions) works:

  `https://en.wikipedia.org/wiki/Convolution`

- Refresh your memory on eigenvectors and eigenvalues:

  `https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors`

Read (optional):

- Work through online tutorials of multi-variable partial derivatives, gradient, divergence, Laplacian, and curl:

  `https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives`

A very brief overview of vector calculus, fluid simulation, and dynamical systems, Part 1

Some Vector Calculus (1)

Gradient (scalar field → vector field)

$$\nabla p = \left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y} \right)$$

- Direction of steepest ascent; magnitude = rate
- *Conservative* vector field: gradient of some scalar (potential) function

Divergence (vector field → scalar field)

$$\nabla \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

- Volume density of outward flux:
  "exit rate: source? sink?"
- *Incompressible/solenoidal/divergence-free vector field*: div u = 0
  can express as curl (next slide) of some vector (potential) function

Laplacian (scalar field → scalar field)

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}$$

- Divergence of gradient
- Measure for difference between point and its neighborhood

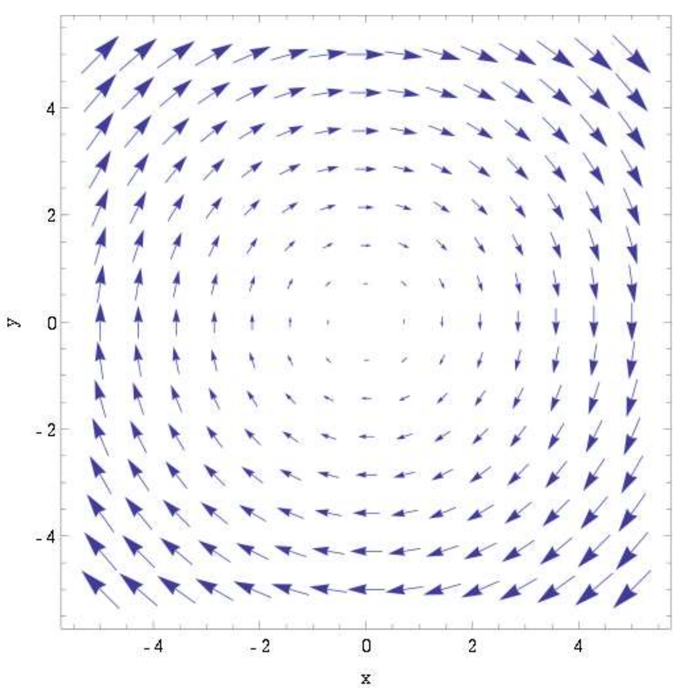
Some Vector Calculus (2)

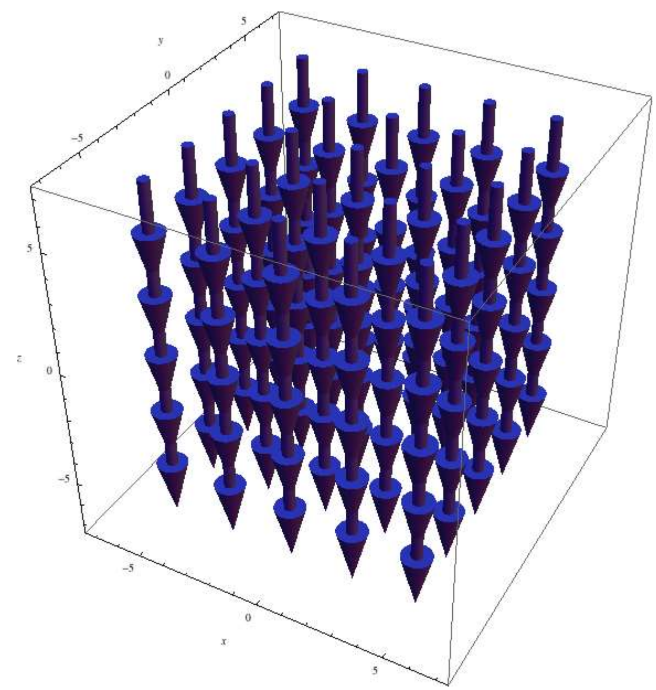
Curl (vector field → vector field)

$$\nabla \times \mathbf{v} = \begin{pmatrix} w_y - v_z \\ u_z - w_x \\ v_x - u_y \end{pmatrix}$$

- Circulation density at a point (*vorticity*)
- If curl vanishes everywhere: irrotational/curl-free field
- Every conservative field is irrotational (and vice versa if simply conn.)

Example:
curl = const
everywhere





Some Vector Calculus (2)

Curl (vector field → vector field)

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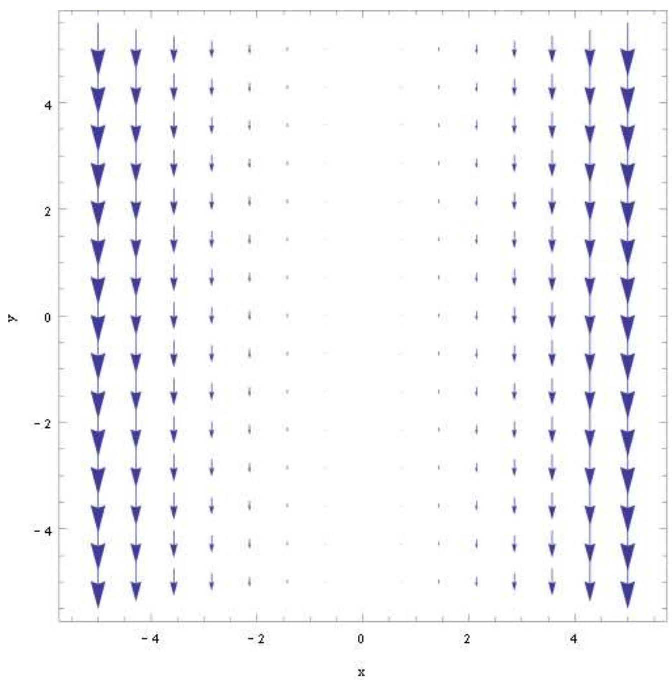
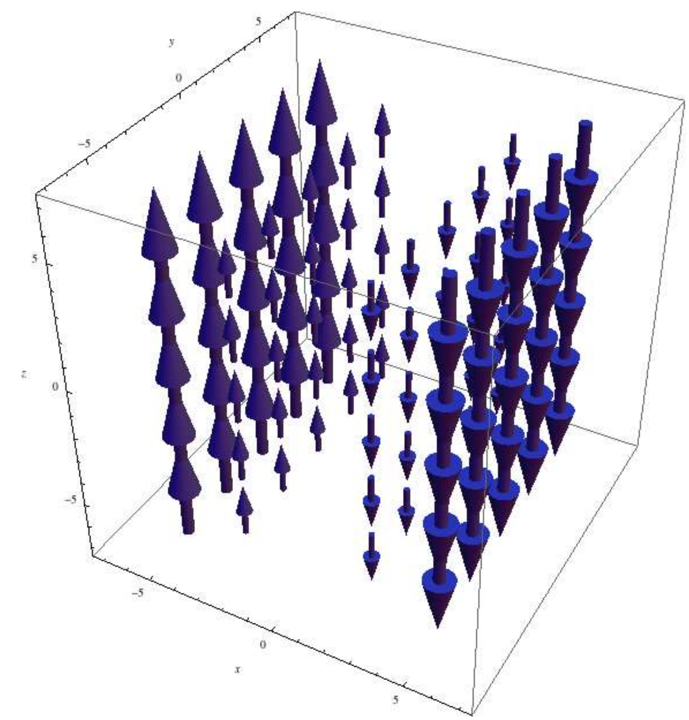
Example: curl not always "obviously rotational"





Some Vector Calculus (2)

Curl (vector field → vector field)

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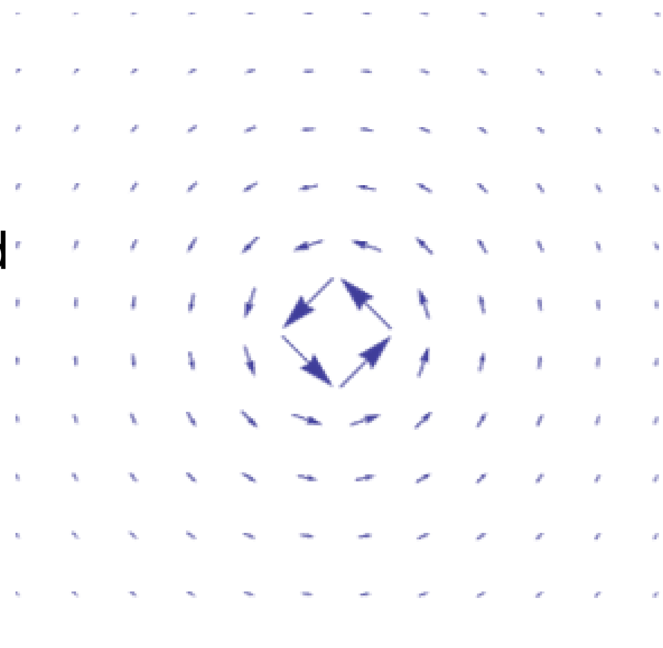
Example:
non-obvious
curl-free field



$$\mathbf{v}(x, y, z) = \frac{(-y, x, 0)}{x^2 + y^2}$$

$$v_x = u_y \qquad \nabla \times \mathbf{v} = 0$$

Velocity gradient $\nabla \mathbf{v}$ is symmetric (see later)

Some Vector Calculus (2)

Curl (vector field → vector field)

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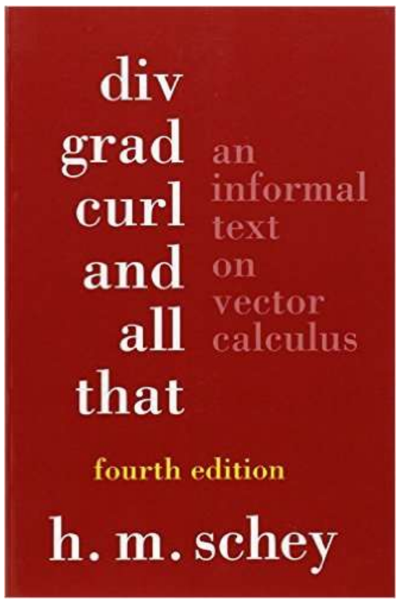
Book:



Interactive tutorial on curl:
`http://mathinsight.org/curl_idea`

Fundamental theorem of vector calculus: any vector field can be expressed as the sum of a solenoidal field and an irrotational field (Helmholtz decomposition)

Thank you.

Thanks for material

- Ronny Peikert
- Helwig Hauser
- Meister Eduard Groeller
- Jens Krüger