AMCS / CS 247 – Scientific Visualization
Lecture 29: Vector Field / Flow Visualization, Pt. 8

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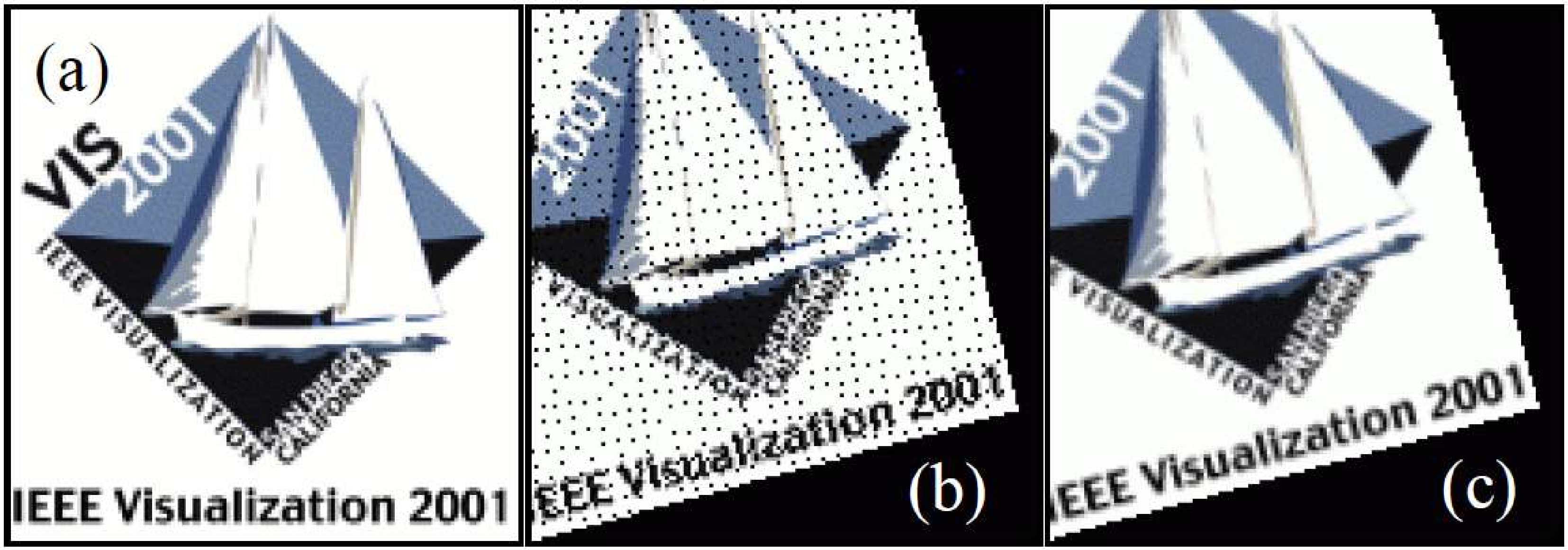
Reading Assignment #15++

Read (required):

- Go over the flow visualization reading assignments again

Lagrangian vs. Eulerian

- Lagrangian: move along with the particle
- Eulerian: consider fixed point in space, look at particles moving through



- Example for pixels: rotate image (a),
  Lagrangian: move pixels forward (b),
  Eulerian: fetch pixels from backward direction (c)

Material Derivative (1)

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$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

Material Derivative (2)

Actually, nothing else than application of the multi-variable chain rule:

$$dT = \frac{\partial T}{\partial t} dt + \frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial z} dz$$

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$$u := \frac{dx}{dt}, \qquad v := \frac{dy}{dt}, \qquad w := \frac{dz}{dt}$$

Material Derivative (2)

Actually, nothing else than application of the multi-variable chain rule:

We are given $T(x, y, z, t)$ with four independent variables;

But now we want to go along a parameterized path with parameter t, so x, y, z become dependent variables: $x(t),\ \ y(t),\ \ z(t)$

Along this path, our goal is now to compute the derivative of the function

$T\left(x(t), y(t), z(t), t\right)$ with t as only independent variable:

$$\frac{d}{dt} T\left(x(t), y(t), z(t), t\right) =$$

$$\frac{\partial}{\partial t} T(x, y, z, t) + \frac{\partial}{\partial x} T(x, y, z, t) \frac{d}{dt} x(t) + \frac{\partial}{\partial y} T(x, y, z, t) \frac{d}{dt} y(t) + \frac{\partial}{\partial z} T(x, y, z, t) \frac{d}{dt} z(t)$$

$$u(t) := \frac{dx(t)}{dt}, \qquad v(t) := \frac{dy(t)}{dt}, \qquad w(t) := \frac{dz(t)}{dt}$$

Advection

Advection equation; velocity field **u(** *x, y, z, t* **)**,
no change following particle, just advection:
set material derivative = 0:

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = 0$$

In the Navier-Stokes equations: "self-advection" of velocity

- Advect scalar components of velocity field individually (actually two equations in 2D, three equations in 3D)

$$\frac{\partial \mathbf{u}}{\partial t} = -\left(\mathbf{u} \cdot \nabla\right) \mathbf{u}$$

Fluid Simulation: Navier Stokes (1)

Incompressible (divergence-free) Navier Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{F},$$

$$\nabla \cdot \mathbf{u} = 0,$$

Components:

- Self-advection of velocity (i.e., advection of velocity according to velocity)
- Pressure gradient (force due to pressure differences)
- Diffusion of velocity due to viscosity (for viscous fluids, i.e., not inviscid)
- Application of (arbitrary) external forces, e.g., gravity, user input, etc.

Fluid Simulation: Navier Stokes (2)

Actually, the momentum equation is a system of equations (2 equations in 2D, 3 equations in 3D)

For 2D:

these are PDEs!

$$\frac{\partial u}{\partial t} = -\left(\mathbf{u} \cdot \nabla\right) u - \frac{1}{\rho}(\nabla p)_x + \nu \nabla^2 u + f_x ,$$

$$\frac{\partial v}{\partial t} = -\left(\mathbf{u} \cdot \nabla\right) v - \frac{1}{\rho}(\nabla p)_y + \nu \nabla^2 v + f_y .$$

Thank you.

Thanks for material

- Ronny Peikert
- Helwig Hauser
- Meister Eduard Groeller
- Jens Krüger