CS 380 - GPU and GPGPU Programming
Lecture 14: GPU Texturing 2

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Reading Assignment #7+8 (until Oct 28)

Read (required):

- Interpolation for Polygon Texture Mapping and Shading,
  Paul Heckbert and Henry Moreton

http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.48.7886

- MIP-Map Level Selection for Texture Mapping

http://ieeexplore.ieee.org/xpl/login.jsp?tp=&arnumber=765326

- OpenGL 4.6 Core Specification,
  Chapter 9 (Frame Buffers and Frame Buffer Objects)

https://www.khronos.org/registry/OpenGL/specs/gl/glspec46.core.pdf

Quiz #2: Oct 17

Organization

- First 30 min of lecture
- No material (book, notes, ...) allowed

Content of questions

- Lectures (both actual lectures and slides)
- Reading assigments
- Programming assignments (algorithms, methods)
- Solve short practical examples

Programming Assignments: Schedule (tentative)

Assignment #1:

- Querying the GPU (OpenGL and CUDA) — due Sep 2

Assignment #2:

- Phong shading and procedural texturing (GLSL) — due Sep 26

Assignment #3:

- Image Processing with (a) GLSL, and (b) CUDA
- Convolutional layers (CUDA) — due Oct 14

Assignment #4:

- Linear Algebra (CUDA) — due Nov 4

Next Lectures

Lecture 15: Wednesday, Oct 17, 13:00 [Quiz #2]

Lecture 16: Thursday, Oct 18, 14:30 ?

next week no lectures!

Lecture 17: Sunday, Oct 28, 13:00 [Quiz #3]

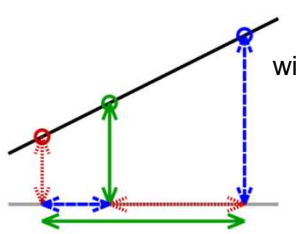
Lecture 18: Wednesday, Oct 31, 13:00

Lecture 19: Thursday, Nov 1, 14:30 ?

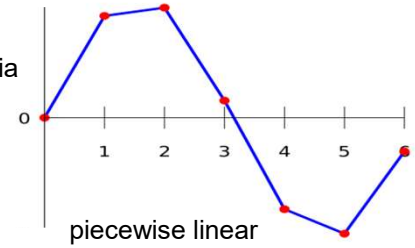
GPU Texturing

Rage / id Tech 5 (id Software)

Linear Interpolation / Convex Combinations



wikipedia



piecewise linear

Linear interpolation in 1D:

$$f(\alpha) = (1-\alpha)v_1 + \alpha v_2$$

Line embedded in 2D (linear interpolation of vertex coordinates/attributes):

$$f(\alpha_1, \alpha_2) = \alpha_1 v_1 + \alpha_2 v_2 \qquad f(\alpha) = v_1 + \alpha(v_2 - v_1)$$
$$\alpha_1 + \alpha_2 = 1 \qquad \alpha = \alpha_2$$

Line segment: $\alpha_1, \alpha_2 \geq 0$ (→ convex combination)

Compare to line parameterization with parameter t:

$$v(t) = v_1 + t(v_2 - v_1)$$

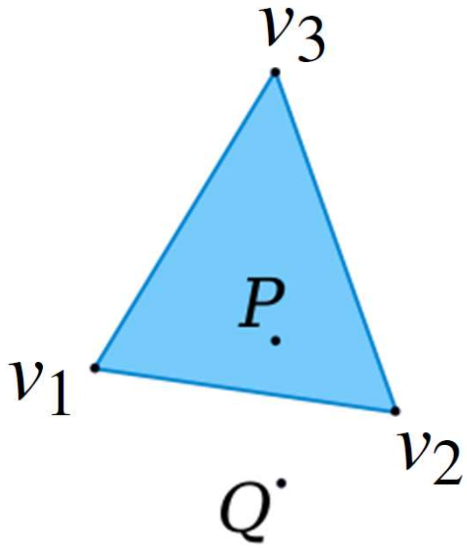
Linear Interpolation / Convex Combinations

Linear combination:

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

Affine combination: restrict to $(n-1)$-dim. subspace:

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$



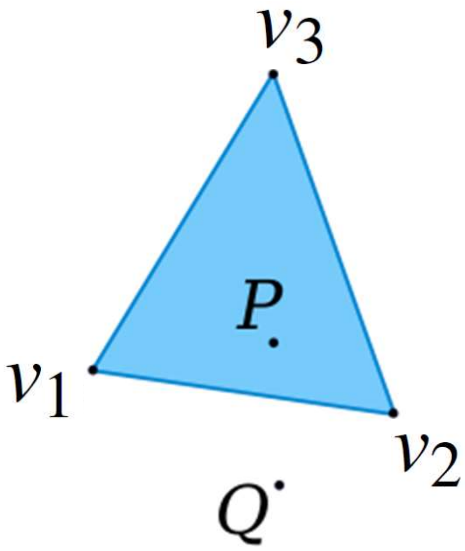
Convex combination: $\alpha_i \geq 0$

(restrict to simplex)

Linear Interpolation / Convex Combinations

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$



Re-parameterize to get affine coordinates:

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 =$$
$$\tilde{\alpha}_1 (v_2 - v_1) + \tilde{\alpha}_2 (v_3 - v_1) + v_1$$
$$\tilde{\alpha}_1 = \alpha_2$$
$$\tilde{\alpha}_2 = \alpha_3$$

Linear Interpolation / Convex Combinations

The weights α_i are the (normalized) barycentric coordinates

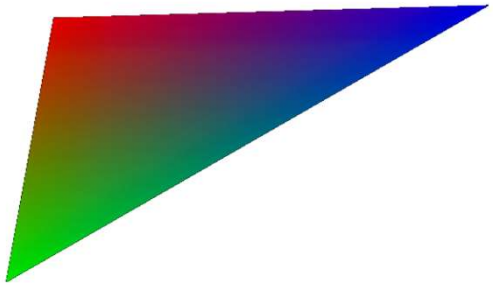
→ linear attribute interpolation in simplex

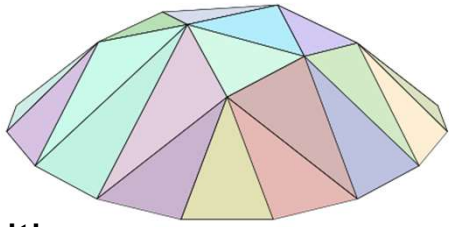
$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$

$$\alpha_i \geq 0$$

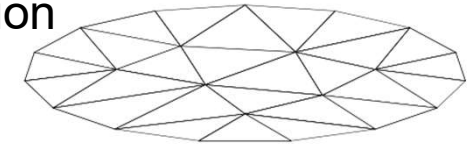
attribute interpolation





spatial position interpolation

wikipedia



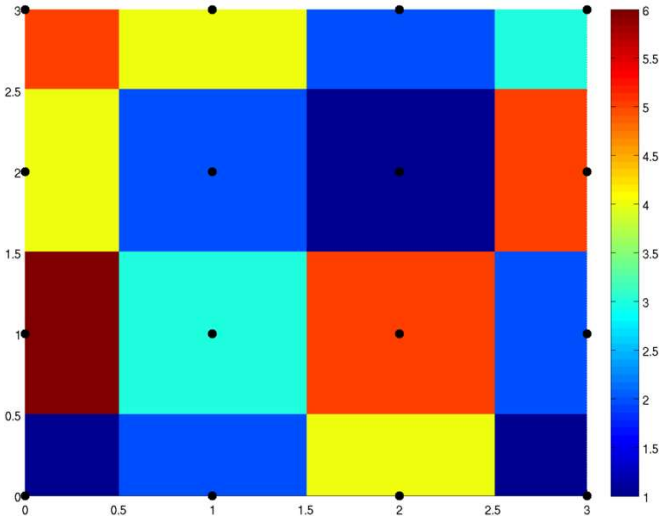
Magnification (Bi-linear Filtering Example)

Original image

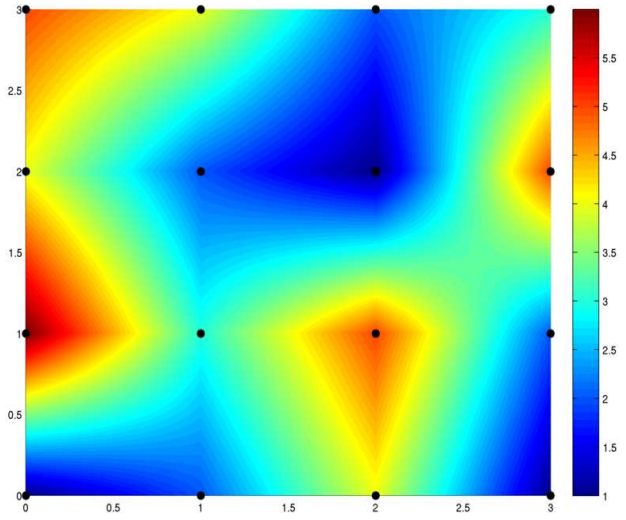
Nearest neighbor

Bi-linear filtering

Nearest-Neighbor vs. Bi-Linear Interpolation

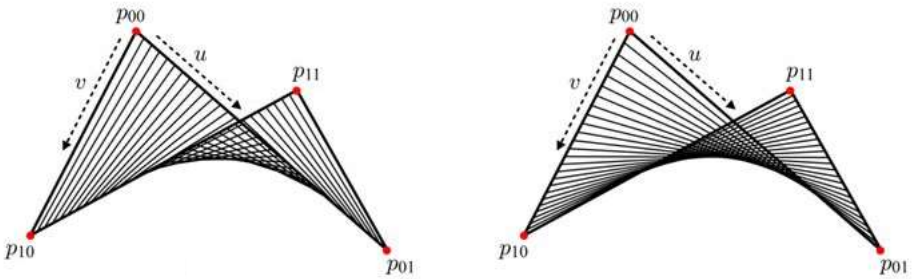


nearest-neighbor



bi-linear

wikipedia



Bilinear patch (courtesy J. Han)

Bi-Linear Interpolation

Consider area between 2x2 adjacent samples (e.g., pixel centers):

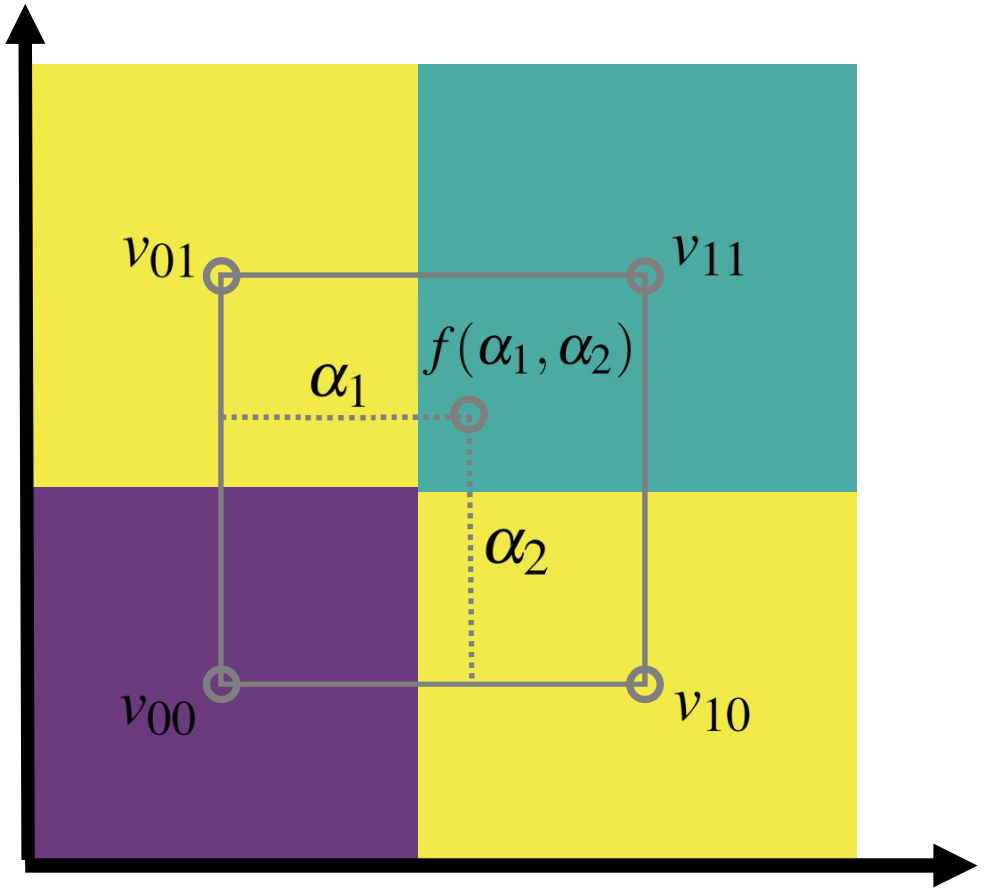
Given any (fractional) position

$$\alpha_1 := x_1 - \lfloor x_1 \rfloor \quad \alpha_1 \in [0.0, 1.0)$$
$$\alpha_2 := x_2 - \lfloor x_2 \rfloor \quad \alpha_2 \in [0.0, 1.0)$$

and 2x2 sample values

$$\begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix}$$

Compute: $f(\alpha_1, \alpha_2)$



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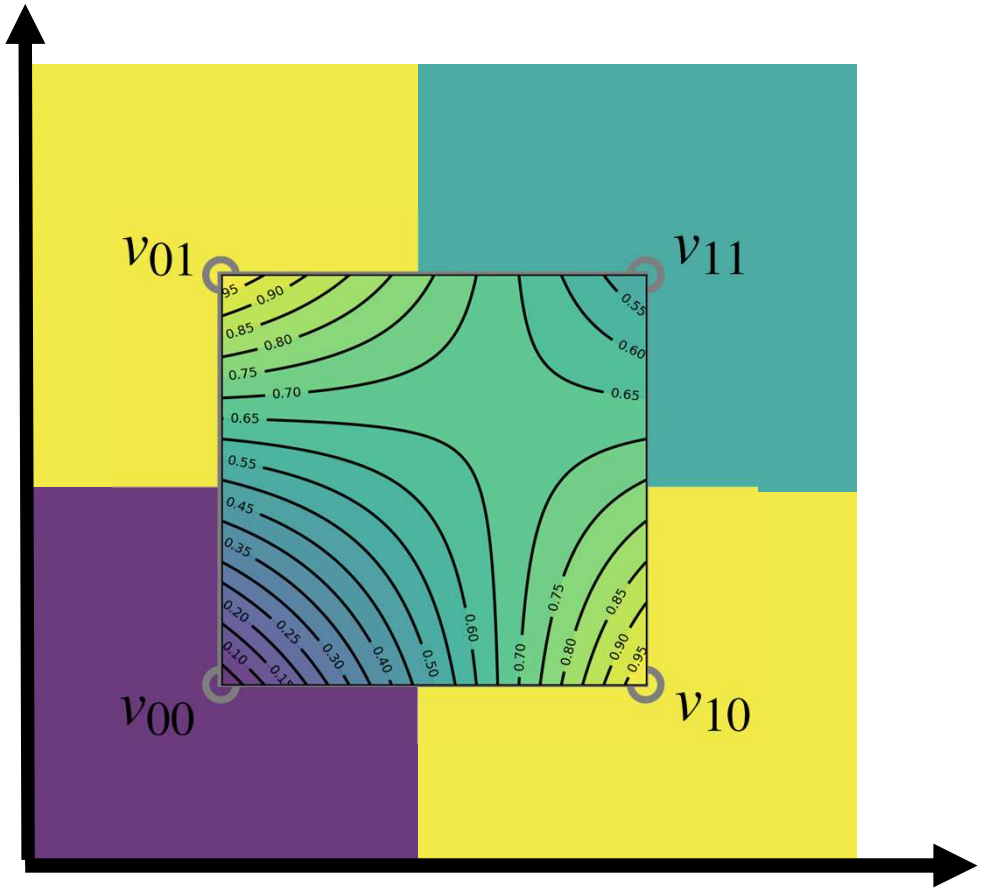

Bi-Linear Interpolation

Weights in 2x2 format:

$$\begin{bmatrix} \alpha_2 \\ (1-\alpha_2) \end{bmatrix} \begin{bmatrix} (1-\alpha_1) & \alpha_1 \end{bmatrix} = \begin{bmatrix} (1-\alpha_1)\alpha_2 & \alpha_1\alpha_2 \\ (1-\alpha_1)(1-\alpha_2) & \alpha_1(1-\alpha_2) \end{bmatrix}$$

Interpolate function at (fractional) position (α_1, α_2) :

$$f(\alpha_1, \alpha_2) = \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

Bi-Linear Interpolation

Interpolate function at (fractional) position (α_1, α_2) :

$$f(\alpha_1, \alpha_2) = \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} (1-\alpha_1)v_{01} + \alpha_1 v_{11} \\ (1-\alpha_1)v_{00} + \alpha_1 v_{10} \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_2 v_{01} + (1-\alpha_2)v_{00} & \alpha_2 v_{11} + (1-\alpha_2)v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

Bi-Linear Interpolation

Interpolate function at (fractional) position (α_1, α_2) :

$$f(\alpha_1, \alpha_2) = \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

$$= (1-\alpha_1)(1-\alpha_2)v_{00} + \alpha_1(1-\alpha_2)v_{10} + (1-\alpha_1)\alpha_2 v_{01} + \alpha_1\alpha_2 v_{11}$$

$$= v_{00} + \alpha_1(v_{10} - v_{00}) + \alpha_2(v_{01} - v_{00}) + \alpha_1\alpha_2(v_{00} + v_{11} - v_{10} - v_{01})$$

REALLY IMPORTANT:

this is a different thing (for a different purpose) than the linear (or, in perspective, rational-linear) interpolation of texture coordinates!!

Thank you.